



RAN - 2103000204023003

**RAN-2103000204023003****S.Y.B.Sc. (Sem - IV) Examination April - 2025****MTH - 403 : Mathematics - X (Old)****Time: 2 Hours ]****[ Total Marks: 50****सूचना : / Instructions**

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नीचे दशावलि निशानीवाणी विगतो उत्तरवली पर अवश्य लभवी.  
**Fill up strictly the details of signs on your answer book**

Name of the Examination:

☛ **S.Y.B.Sc. (Sem - IV)**

Name of the Subject :

☛ **MTH - 403 : Mathematics - X (Old)**Subject Code No.: **2103000204023003**

Seat No.:

Student's Signature

**Q.1. Answer any Five from the following.****10**

1. Write the definition of one-one function with illustration
2. Let  $f(x) = \sin x$  then find the value of  $f\left[0, \frac{\pi}{6}\right]$  then  $f\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$  and  $f\left[0, \frac{\pi}{2}\right]$
3. If  $A$  and  $B$  are countable sets then prove that  $A \cup B$  is also countable.
4. Write the definition of bounded set.
5. Let  $\{S_n\}_{n=1}^{\infty}$  be the sequence defined by  
 $S_1 = 1, S_2 = 1$  and  $S_{n+1} = S_n + S_{n-1}$  ( $n \geq 2$ ) then  $S_7$
6. Write the definition of Divergent Sequence with illustration.
7. State the division algorithm.
8. Define the greatest common divisor.

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**Q.2 Answer any Two from the following.****10**

1. Let  $f(x) = \sin x$  ( $0 \leq x \leq 2\pi$ ) and  $g(x) = \sqrt{1 - \cos^2 x}$  ( $-\infty < x < \infty$ ). For the this pair of functions  $f$  and  $g$ , is  $g$  is an extension of  $f$ ?
2. Let  $f(x) = \tan x$  ( $-\frac{\pi}{2} < x < \frac{\pi}{2}$ )
  - (i) What is the domain of  $f$ ?
  - (ii) What is the range of  $f$ ?
  - (iii) Let  $A = [-\frac{\pi}{2}, -\frac{\pi}{4}]$ ,  $B = [\frac{\pi}{4}, \frac{\pi}{2}]$ . Does  $f(A \cap B) = f(A) \cap f(B)$ ?
3. If  $f: A \rightarrow B$  and  $X \subset A$ ,  $Y \subset A$  then prove that  $f(X \cap Y) \subset f(X) \cap f(Y)$ .  
Also give an example to show that  $f(X \cap Y)$  and  $f(X) \cap f(Y)$  need not be equal

**Q.3 Answer any Two from the following.****10**

1. Find least upper bound and greatest lower bound for following sets.
  - i.  $A = \{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \dots\}$
  - ii.  $B = \{\pi + 1, \pi + 2, \pi + 3, \dots\}$
  - iii.  $C = \{\pi + 1, \pi + \frac{1}{2}, \pi + \frac{1}{3}, \dots\}$
2. If  $A_1, A_2, A_3, \dots$  are countable sets then prove that  $\bigcup_{n=1}^{\infty} A_n$  is countable.
3. Check the convergence of following sequences,
  - i.  $\{n \sin \frac{\pi}{n}\}_{n=1}^{\infty}$  ,      ii.  $\{\sin(n\theta\pi)\}_{n=1}^{\infty}$
  - iii.  $\{n^3 e^n\}_{n=1}^{\infty}$

**Q.4 Answer any Two from the following.****10**

1. If the sequence  $\{S_n\}_{n=1}^{\infty}$  of real numbers is convergent and  $S_n \leq M$  and if  $\{S_n\}_{n=1}^{\infty}$  converges to  $L$  then  $L \leq M$ .

2. If  $\{S_n\}_{n=1}^{\infty}$  is convergent and converges to  $L$  then prove that  $\{|S_n|\}_{n=1}^{\infty}$  converges to  $|L|$ .
3. If the sequence  $\{S_n\}_{n=1}^{\infty}$  of real numbers is convergent and converges to  $L$  then  $\{S_n\}_{n=1}^{\infty}$  cannot converge to a limit distinct from  $L$ .

**Q.5 Answer any Two from the following.**

**10**

1. Use Euclidean Algorithm to find the values of  $x$  and  $y$  from following,
  - i.  $\gcd(1769, 2378) = 1769x + 2378y$
  - ii.  $\gcd(12378, 3054) = 12378x + 3054y$
2. Prove that  $ab = ac \pmod{n}$  if and only if  $b = c \pmod{\frac{n}{d}}$  where  $d$  is greatest common divisor of  $a$  and  $n$ .
3. Prove that if  $\gcd(a, b) = d$  then there exist integers  $x$  and  $y$  such that  $d = ax + by$ .

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